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DISTINCTIVE FEATURES OF COMBUSTION OF A PROPELLANT AT A LEWIS NUMBER OTHER THAN UNITY IN THE GAS PHASE

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The nonstationary combustion regimes of a homogeneous solid propellant, i.e., stability and steady-state combustion in an acoustic-wave field, are investigated. Analytical formulas for the boundary of stable combustion, the nonstationary rate of combustion, and the acoustic admittance are found. Account for the relaxation time of the gas phase substantially changes the concept of the region of stable combustion of a solid propellant. The exothermal reaction of decomposition of the propellant turns out to be improbable by virtue of the smallness of the stability region and the positiveness of phenomenological coefficients. In the case of pressure oscillations with a period, comparable to the relaxation time of the gas phase, the acoustic admittance of the propellant depends strongly on both this relaxation time and the Lewis number in the gas phase.

In designing solid-propellant rocket engines, one must know the manner in which the combustion of a propellant charge occurs under nonstationary conditions. These conditions can be created artificially, for example, in controlling the engine thrust. Another type of nonstationary processes occurs when the combustion stability is lost. In the theory of rocket engines, one recognizes low-frequency and high-frequency instabilities [1]. At the present time, the first instability has been studied more thoroughly than the second one. This is attributed to the fact that at low-frequency oscillations the relaxation time t_g of the gas phase can be disregarded by assuming it to be equal to zero. The important quantity for practical implementation, i.e., the nonstationary linear rate $u(t)$ of combustion of the propellant, is determined by slow thermal processes in the solid propellant itself; these processes are characterized by the relaxation time t_c ($t_c >> t_g$). This circumstance substantially simplifies a theoretical analysis of low-frequency processes.

Figure 1 (top) illustrates one of the simplest variants of a solid-propellant rocket engine and a scheme of decomposition of a homogeneous propellant (bottom) which is widely used in theoretical investigations [2, 3]. In the space $x < x_s(t)$, the temperature T_c of the propellant increases from the initial T_0 to T_c. On the surface $x_s(t)$, the primary decomposition reaction with a thermal effect L occurs during which the solid propellant is decomposed into a gaseous combustible and an oxidizer. This mixture is characterized by the temperature T that in the so-called heating area grows from T_s to the value of T_b. The subsequent chemical reaction with a thermal effect Q and a base pressure p_0 proceeds on the surface $x_f(t)$, behind which the constant flame temperature T_b is established in the stationary regime of combustion. The region $x > x_f(t)$ is called the flame area.

In what follows, just as in the similar investigations performed earlier [4–6], the thickness of the chemical-reaction zone for $x = x_s(t)$ and $x = x_f(t)$ is considered to be infinitely thin.

The leading edge $x_f(t)$ of the flame in the gas phase is usually located so close to the surface $x_s(t)$ of the propellant that on the interval $\Delta x = x_f(t) - x_s(t)$ the projection of the vector $\mathbf{v}$ of the gas velocity onto the x axis is much higher than the projection onto the y axis. Moreover, the scales of diffusion-thermal processes

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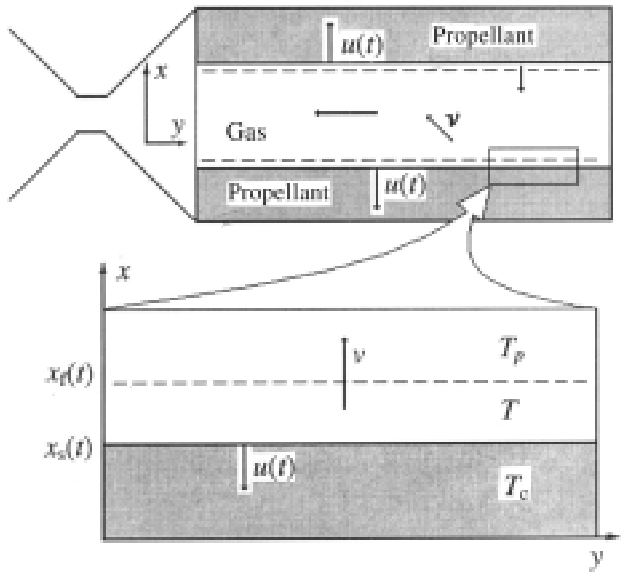

Fig. 1. Scheme of a propellant rocket engine (top) and model of combustion of a solid propellant (bottom).

are much smaller, as a rule, than the dimensions of a propellant charge. All this makes it possible to make the following assumptions:

(1) the diffusion-thermal processes in the solid and gas phases can be considered in the approximation which is one-dimensional in space (i.e., along the x axis);

(2) the propellant charge and the flame itself extend to infinity;

(3) the pressure field for these processes can be considered to be independent of any spatial variables, except for those cases where the wavelength of acoustic oscillations is comparable to Δx.

Additional assumptions presuppose the ideality of the gas, the constancy of the molecular weight of the gas, and the equality of the heat capacities of the solid and gas phases [4–6].

Then, under these assumptions, the problem of finding the nonstationary combustion rate of the solid propellant is strongly simplified, although it still remains rather complex for analytical investigation. Nevertheless, it is possible to solve the problem of finding the region of stable combustion with one-dimensional disturbances and determining the nonstationary combustion rate at a pressure varying harmonically with time with a small amplitude [4–6].

The investigations carried out in these trends are of importance in view of the above-mentioned high-frequency instability. Although modern propellants are mainly miscible ones and contain a metal (usually, aluminum), this fact by no means minimizes the importance of investigating the combustion of homogeneous propellants as simpler systems of the initial stage with subsequent transition to more complex systems, such as miscible propellants. On the other hand, it is possible that the development of chemistry and technology will produce in the future more efficient homogeneous propellants that will be as good as miscible propellants in thermodynamic characteristics.

1. Mathematical Formulation of the Problem. In [4, 5, 7, 8], cases are investigated where evaporation on the propellant surface $x_s(t)$ follows the Clausius–Clapeyron law. In [4, 5, 8], the Lewis number Le, i.e., the ratio of the diffusion coefficient D to the thermal diffusivity κ of the gas, was assumed to be equal to unity. The more complex process with Le $\neq 1$ is studied in [7], where, as has been found, the Lewis number affects strongly the value of the nonstationary combustion rate $u(t) = dx_s(t)/dt$.

The reaction of pyrolysis can also proceed on the surface $x_s(t)$. In this case, the combustion rate of the solid propellant depends explicitly on the temperature T_s of the propellant surface and the pressure p: u

$= u(T_s, p)$. According to the ideas of F. A. Williams [2], the decomposition of the propellant into gaseous components occurs simultaneously by both evaporation and pyrolysis. Depending on the pressure, both mechanisms of decomposition are the limiting cases of the general mechanism. Pyrolysis becomes predominant over evaporation at relatively low pressures. However, the threshold value of the pressure p_*, arbitrarily separating different mechanisms of decomposition, "is unique" for each type of propellant.

Below we consider the arbitrary values of the Le number and its influence on the process of combustion of a solid propellant gasified by the pyrolysis reaction.

The above physical pattern of combustion of a homogeneous propellant is mathematically described by the system of equations [6]

$$-\infty < x < x_s(t): \quad \rho_c c_c \frac{\partial T_c}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_c \frac{\partial T_c}{\partial x}\right);$$

$$x_s(t) < x < x_f(t): \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho v = 0, \quad \rho\left(\frac{\partial Y}{\partial t} + v\frac{\partial Y}{\partial x}\right) = \frac{\partial}{\partial x}\left(D\rho\frac{\partial Y}{\partial x}\right),$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \rho c_p \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{dp}{dt}; \quad x_f(t) < x < +\infty: \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho v_b = 0, \tag{1}$$

$$\rho c_p\left(\frac{\partial T_b}{\partial t} + v_b\frac{\partial T_b}{\partial x}\right) = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T_b}{\partial x}\right) + \rho c_p \frac{\gamma-1}{\gamma}\frac{T_b}{p}\frac{dp}{dt}, \quad p = \rho R T.$$

The boundary conditions are as follows:

$$x \to -\infty: \quad T_c = T_0,$$

$$x = x_s(t): \quad -\rho_c\frac{dx_s}{dt} = -\rho\frac{dx_s}{dt} + \rho v, \quad -\rho_c\frac{dx_s}{dt} = -\rho\frac{dx_s}{dt} + \rho v Y - D\rho\frac{\partial Y}{\partial x},$$

$$-\rho_c\frac{dx_s}{dt} = m_1(T, p), \quad T = T_c, \quad \lambda_c\frac{\partial T_c}{\partial x} = \lambda\frac{\partial T}{\partial x} + L\rho_c\frac{dx_s}{dt};$$

$$x = x_f(t): \quad Y = 0, \quad v = v_b, \quad T = T_b, \quad \lambda\frac{\partial T}{\partial x} = \lambda\frac{\partial T_b}{\partial x} - D\rho Q\frac{\partial Y}{\partial x}, \quad -D\rho\frac{\partial Y}{\partial x} = m_2(T, p);$$

$$x \to +\infty: \quad |T_b| < +\infty.$$

The sign before the L, taken in the boundary conditions, corresponds to the endothermal reaction. The mass combustion rates m_1 and m_2 are equal to each other only in the case where the combustion is stable.

The dependences of the diffusion coefficient and the thermal conductivity of the gas on the temperature and pressure will be taken in the form

$$D \sim \frac{T^2}{p}, \quad \lambda \sim T. \tag{2}$$

They are close to those observed experimentally [9] in form.

Assigning the superscript 0 to the stationary values of the symbols, we pass in the system of equations (1) to the dimensionless quantities and to the Lagrangian coordinate ξ according to the expressions

$$-\infty < x < x_s(t):\ \xi = \frac{u^0}{\kappa_c}[x - x_s(t)],\quad u^0 \equiv -\left(\frac{dx_s}{dt}\right)^0;$$

$$x_s(t) < x < +\infty:\ \xi = \frac{u^0}{\sigma\kappa_c\,\rho_c}\int\limits_{x_s(t)}^{x}\rho(y,t)\,dy,\quad \kappa_c = \frac{\lambda_c}{c_c\,\rho_c},$$

$$\theta_c = \frac{T_c}{T_s^0},\quad \theta = \frac{T}{T_s^0},\quad \theta_b = \frac{T_b}{T_s^0},\quad \theta_0\frac{T_0}{T_s^0},\quad q = \frac{Q}{c_pT_s^0},\quad \sigma = \frac{D\,(\rho^0)^2}{\kappa_c\,\rho_c^2}, \tag{3}$$

$$B = -\frac{\rho_c}{m_1^0}\frac{dx_s}{dt},\quad l = \frac{L}{c_cT_s^0},\quad \tau = \frac{(u^0)^2}{\kappa_c}t,\quad \eta = \frac{p}{p_0},\quad m_1^0 = m_2^0 = \rho_c u^0 .$$

This allows us to separate the hydrodynamic part of the problem from the diffusion-thermal part. Using the second formula from Eq. (2) and the equation of state of an ideal gas, we can easily verify that

$$\frac{\lambda\rho}{\lambda^0\rho^0} = \eta . \tag{4}$$

The partial derivatives in Eq. (1) are determined in an Euler coordinate system. Therefore, these derivatives must be redetermined in a new Lagrangian coordinate system. To distinguish the time derivatives in the indicated coordinate systems, we introduce the subscript Eu for the Euler derivative. However, the required calculations are given in detail only for the temperature T, since for the remaining parameters the transition to the new coordinate system will not be difficult. Thus:

$$\left.\frac{\partial T}{\partial t}\right|_{\mathrm{Eu}} = \frac{\partial T}{\partial t} + \frac{\partial\xi}{\partial t}\frac{\partial T}{\partial\xi},\quad \frac{\partial\xi}{\partial t} = -\frac{u^0\rho}{\sigma\kappa_c\,\rho_c}\frac{dx_s}{dt} + \frac{u^0}{\sigma\kappa_c\,\rho_c}\int\limits_{x_s(t)}^{x}\frac{\partial\rho}{\partial t}\,dy .$$

Substitution into this expression of the quantity $\partial\rho/\partial t$ from the second expression in Eq. (1) and subsequent evaluation of the integral give

$$\frac{\partial\xi}{\partial t} = -\frac{u^0}{\sigma\kappa_c\,\rho_c}\left(\rho\frac{dx_s}{dt} + \rho v - \rho v\,\big|_{x=x_s(t)}\right) = -\frac{u^0}{\sigma\kappa_c\,\rho_c}\left(\rho_c\frac{dx_s}{dt} + \rho v\right).$$

Having calculated the derivative of ξ with respect to x

$$\frac{\partial\xi}{\partial x} = \frac{u^0\rho}{\sigma\kappa_c\,\rho_c},$$

it is easy to transform the left-hand side of the equation for T:

$$\left.\frac{\partial T}{\partial t}\right|_{\mathrm{Eu}} + v\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t} - \frac{u^0}{\sigma\kappa_c\,\rho_c}\left(\rho_c\frac{dx_s}{dt} + \rho v\right)\frac{\partial T}{\partial\xi} + v\frac{\partial\xi}{\partial x}\frac{\partial T}{\partial\xi} =$$

$$= \frac{\partial T}{\partial t} - \frac{u^0}{\sigma\kappa_c}\frac{dx_s}{dt}\frac{\partial T}{\partial\xi} = \frac{(u^0)^2}{\sigma\kappa_c}\left(\sigma\frac{\partial T}{\partial\tau} - \frac{1}{u^0}\frac{dx_s}{dt}\frac{\partial T}{\partial\xi}\right).$$

The transition to the Lagrangian coordinate ξ in the term, responsible for conductive heat transfer, is somewhat simpler:

$$\frac{1}{\rho c_p}\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)=\frac{\lambda\rho}{c_p}\left(\frac{u^0}{\sigma\kappa_c\,\rho_c}\right)^2\frac{\partial^2 T}{\partial\xi^2}=\frac{(u^0)^2}{\sigma\kappa_c}\,\eta\,\frac{\partial^2 T}{\partial\xi^2}\,.$$

Here we used formula (4), the form of the parameter σ from Eq. (3), and the above assumption $c_c = c_p$.

Further transition to the Lagrangian coordinate and to the dimensionless quantities in the remaining equations is not difficult. When transforming the diffusion equation, it must be taken into account that by virtue of the above dependences (2) for D and λ, the following equalities hold:

$$\frac{D\rho^2}{\sigma\kappa_c\,\rho_c^2}=\frac{D\rho^2 c_p}{\lambda^0\rho^0}=\mathrm{Le}\,\eta\,,\quad \mathrm{Le}=\frac{D^0\rho^0 c_p}{\lambda^0}=\mathrm{const}\,.$$

The mathematical formulation of the problem in the new variables has the form

$$-\infty<\xi<0:\ \frac{\partial\theta_c}{\partial\tau}+B\frac{\partial\theta_c}{\partial\xi}-\frac{\partial^2\theta_c}{\partial\xi^2}=0\,,$$

$$0<\xi<\xi_f:\ \sigma\frac{\partial\theta}{\partial\tau}+B\frac{\partial\theta}{\partial\xi}-\eta\frac{\partial^2\theta}{\partial\xi^2}=\sigma\Gamma\frac{\theta}{\eta}\frac{d\eta}{d\tau}\,,\quad \sigma\frac{\partial Y}{\partial\tau}+B\frac{\partial Y}{\partial\xi}-\eta\,\mathrm{Le}\frac{\partial^2 Y}{\partial\xi^2}=0\,, \tag{5}$$

$$\xi_f<\xi<+\infty:\ \sigma\frac{\partial\theta_b}{\partial\tau}+B\frac{\partial\theta_b}{\partial\xi}-\eta\frac{\partial^2\theta_b}{\partial\xi^2}=\sigma\Gamma\frac{\theta_b}{\eta}\frac{d\eta}{d\tau}\,,\quad B=B\,(t)\,,\quad \xi_f=\xi_f\,(t)\,,\quad \Gamma=\frac{\gamma-1}{\gamma}\,.$$

The boundary conditions are as follows:

$$\xi\to-\infty:\ \theta_c=\theta_0\,,$$

$$\xi=0:\ \theta_c=0\,,\quad \frac{\partial\theta_c}{\partial\xi}=\eta\frac{\partial\theta}{\partial\xi}-lB\,,\quad B\,(1-Y)+\eta\,\mathrm{Le}\frac{\partial Y}{\partial\xi}=0\,,\quad B=m_1\,(\theta,\eta)/m_1^0\,,$$

$$\xi=\xi_f:\ \theta=\theta_b\,,\quad \frac{\partial\theta}{\partial\xi}=\frac{\partial\theta_b}{\partial\xi}-q\,\mathrm{Le}\frac{\partial Y}{\partial\xi}\,,\quad -\eta\,\mathrm{Le}\frac{\partial Y}{\partial\xi}=m_2\,(\theta,\eta)/m_2^0\,,\quad Y=0\,,$$

$$\xi\to+\infty:\ \left|\theta_b\right|<+\infty\,.$$

In the system of equations (5), it is the dimensionless relaxation time σ of the diffusion-thermal processes in the gas that is the small parameter: $\sigma \ll 1$. The equation of state of the gas is not given here, since this equation is required for solving the hydrodynamic part of the problem. For further analysis, it is necessary to find stationary solutions of system (5). This can be done easily; therefore, we present the solutions sought without preliminary calculations:

$$\theta_c^0=\theta_0+(1-\theta_0)\exp\xi\,,\quad \theta^0=\theta_0-l+(1-\theta_0+l)\exp\xi\,,\quad Y=1-(1-Y_s^0)\exp(\xi/\mathrm{Le})\,,$$

$$\theta_b^0=\theta_0+q-l\,,\quad B^0=1\,,\quad \xi_f^0=-\mathrm{Le}\ln(1-Y_s^0)=-\ln\frac{q}{1+l-\theta_0}\,,\quad Y_s^0=1-\left(\frac{1-\theta_0+l}{q}\right)^{1/\mathrm{Le}}\,,$$

where the notation Y_s^0 is introduced for the mass concentration of the combustible gas near the surface of the propellant decomposition (in stationary combustion).

2. Equations for a Weakly Disturbed System. In this section and in what follows, the pressure oscillations are considered to be harmonic and small in amplitude. The solution of problem (1) or (5) with such a law of change of the pressure is used for calculating the acoustic admittance [8, 10].

To find the region of stable combustion and the nonstationary combustion rate of the propellant at variable pressure, it is necessary to investigate the processes that correspond to the solutions of Eqs. (5) of the form

$$\theta_c = \theta_c^0 + \vartheta_c(\xi)\exp(\Omega\tau)\,, \quad \theta = \theta^0 + \vartheta(\xi)\exp(\Omega\tau)\,, \quad Y = Y^0 + y(\xi)\exp(\Omega\tau)\,,$$
$$\theta_b = \theta_b^0 + \vartheta_b(\xi)\exp(\Omega\tau)\,, \quad \xi_f = \xi_f^0 + s\exp(\Omega\tau)\,, \quad B = 1 + b\exp(\Omega\tau)\,, \quad \eta = 1 + \phi\exp(\Omega\tau)\,, \tag{6}$$

where Ω is the increment in the buildup of disturbances, whereas the supplements to the stationary solutions are assumed to be small quantities of first order. Substituting Eqs. (6) into Eqs. (5) and retaining only the terms which are linear in disturbance, we obtain the system of equations

$$-\infty < \xi < 0: \ \frac{d^2\vartheta_c}{d\xi^2} - \frac{d\vartheta_c}{d\xi} - \Omega\vartheta_c = \Delta b\exp\xi\,, \quad \Delta = 1 - \theta_0\,,$$

$$0 < \xi < \xi_f^0: \ \frac{d^2\vartheta}{d\xi^2} - \frac{d\vartheta}{d\xi} - \sigma\Omega\vartheta = [b - (1 + \sigma\Omega\Gamma)\,\phi]\,(\Delta + l)\exp\xi - \sigma\Omega\Gamma\,(\theta_0 - l)\,\phi\,,$$

$$\mathrm{Le}\,\frac{d^2y}{d\xi^2} - \frac{dy}{d\xi} - \sigma\Omega y = \frac{a}{\mathrm{Le}}\,(\phi - b)\exp(\xi/\mathrm{Le})\,, \quad a = 1 - Y_s^0\,,$$

$$\xi_f^0 < \xi < +\infty: \ \frac{d^2\vartheta_b}{d\xi^2} - \frac{d\vartheta_b}{d\xi} - \sigma\Omega\vartheta_b = -\sigma\Omega\Gamma\theta_b^0\phi\,,$$

$$\xi \to -\infty: \ \vartheta_c \to 0\,,$$

$$\xi = 0: \ \vartheta_c = \vartheta\,, \quad y - \mathrm{Le}\,\frac{dy}{d\xi} = a\,(b - \phi)\,, \quad b = \frac{1}{\Delta}\,\frac{k}{r}\,\vartheta + \left(\nu - \frac{\mu k}{r}\right)\phi\,,$$

$$\frac{d\vartheta_c}{d\xi} = \frac{d\vartheta}{d\xi} + (\Delta + l)\,\phi - lb\,,$$

$$\xi = \xi_f^0: \ \vartheta = \vartheta_b - qs\,, \quad \frac{d\vartheta}{d\xi} = \frac{d\vartheta_b}{d\xi} - \mathrm{Le}\,q\,\frac{dy}{d\xi} + \frac{1 - \mathrm{Le}}{\mathrm{Le}}\,qs\,, \quad y = \frac{1}{\mathrm{Le}}\,s\,, \tag{7}$$

$$\frac{1}{\mathrm{Le}}\,s - \mathrm{Le}\,\frac{dy}{d\xi} = \frac{k}{\Delta}\,\vartheta_b + (\nu - 1)\,\phi\,,$$

$$\xi \to +\infty: \ |\vartheta_b| < +\infty\,.$$

The parameters k, r, μ, and ν are called phenomenological coefficients [10, 11]. They are determined from experiment according to the formulas

$$k = (T_s - T_0)\left(\frac{\partial \ln m}{\partial T_0}\right)_p, \quad r = \left(\frac{\partial T_s}{\partial T_0}\right)_p, \quad \nu = \left(\frac{\partial \ln m}{\partial \ln p}\right)_{T_0}, \quad \mu = \frac{1}{T_s - T_0}\left(\frac{\partial T_s}{\partial \ln p}\right)_{T_0}.$$

The physical premises for introduction of these coefficients into the theory of nonstationary combustion are two functional dependences for the mass combustion rate m of propellants. The first of these is experimental: $m = m(p, T_0)$. The second one is to a greater degree theoretical and has the form $m = m(p, T_s)$. Since both formulas hold for the same propellant, according to Ya. B. Zel'dovich we have a unique transition from the (p, T_s) plane to the (p, T_0) plane which is more convenient from the practical viewpoint [11]. Therefore, in the fifth boundary condition of system (5) we carried out the following transformations:

$$\frac{m_1(T, p)}{m_1^0} = 1 + \frac{1}{m_1^0}\left(\frac{\partial m_1}{\partial T_s}\right)_p \delta T_s + \frac{1}{m_1^0}\left(\frac{\partial m_1}{\partial p}\right)_{T_s} \delta p \approx$$

$$\approx 1 + \frac{1}{m_1^0}\left(\frac{\partial m_1^0}{\partial T_s^0}\right)_p \delta T_s + \frac{1}{m_1^0}\left(\frac{\partial m_1^0}{\partial p}\right)_{T_s} \delta p = 1 + \left(\frac{\partial \ln m_1^0}{\partial T_s^0}\right)_p \delta T_s + \left(\frac{\partial \ln m_1^0}{\partial \ln p}\right)_{T_s} \delta \ln p\,;$$

$$\left(\frac{\partial \ln m_1^0}{\partial T_s^0}\right)_p = \left(\frac{\partial \ln m_1^0}{\partial T_0}\right)_p \left(\frac{\partial T_s^0}{\partial T_0}\right)_p^{-1} \} = \frac{1}{T_s^0 - T_0}\frac{k}{r},$$

$$\left(\frac{\partial \ln m_1^0}{\partial \ln p}\right)_{T_s} = \frac{\partial(\ln m_1^0, T_s^0)}{\partial(\ln p, T_s^0)}\frac{\partial(\ln p, T_0)}{\partial(\ln p, T_0)} = \left(\frac{\partial T_s^0}{\partial T_0}\right)_p^{-1} \frac{\partial(\ln m_1^0, T_s^0)}{\partial(\ln p, T_0)} = \nu - \frac{\mu k}{r},$$

as a result of which from Eq. (7) we obtained the fourth boundary condition. In a similar manner we write the ratio m_2/m_2^0. Therefore, the transformation of the eighth boundary condition from Eq. (5) is not given here.

The differential equations in (7) are simple and the determination of their solutions is not difficult. Because of this, they will be presented below without preliminary calculations. We only note that in the forms of the functions ϑ_c and ϑ_b, account is taken of the boundary conditions for $x = -\infty$ and $x = +\infty$ respectively:

$$\vartheta_c = Aq\exp(z\xi) - \frac{b\Delta}{\Omega}\exp\xi,$$

$$\vartheta = Fq\exp(z_1\xi) + Gq\exp(z_2\xi) + \frac{\Delta + l}{\sigma\Omega}[\phi(1 + \sigma\Omega\Gamma) - b]\exp\xi + \Gamma(\theta_0 - l)\phi,$$

$$y = C\exp(z_3\xi/\mathrm{Le}) + D\exp(z_4\xi/\mathrm{Le}) + \frac{ab}{\sigma\Omega\,\mathrm{Le}}(b - \phi)\exp(\xi/\mathrm{Le}), \quad \vartheta_b = Hq\exp(z_2\xi) + \Gamma\theta_b^0\phi,$$

$$z = \frac{1}{2}(1 + \sqrt{1 + 4\Omega}), \quad z_1 = \frac{1}{2}(1 + \sqrt{1 + 4\sigma\Omega}), \quad z_2 = \frac{1}{2}(1 - \sqrt{1 + 4\sigma\Omega}),$$

$$z_3 = \frac{1}{2}\,(1 + \sqrt{1 + 4\sigma\Omega\,\mathrm{Le}}\,)\,, \quad z_4 = \frac{1}{2}\,(1 - \sqrt{1 + 4\sigma\Omega\,\mathrm{Le}}\,)\,.$$

Using the given solutions in the boundary conditions $\xi = 0$ and $\xi = \xi_f$ of the system of equations (7), we obtain algebraic equations with the unknown constants of integration A, b, F, G, C, D, H, and s:

$$\begin{gathered}
A + \frac{a^{\mathrm{Le}}}{\Omega}\left(\frac{1}{\sigma} - \delta\right)b - F - G = \left(\frac{a^{\mathrm{Le}}}{\sigma\Omega} + \frac{\Gamma}{q}\right)\phi\,, \\
Az + a^{\mathrm{Le}}\left(\frac{1}{\sigma\Omega} + 1 - \delta - \frac{\delta}{\Omega}\right)b - Fz_1 - Gz_2 = a^{\mathrm{Le}}\left(\frac{1}{\sigma\Omega} + 1 + \Gamma\right)\phi\,, \\
ab - Cz_4 - Dz_3 = a\phi\,, \\
-\frac{1}{\delta a^{\mathrm{Le}}}\frac{k}{r}A + \left(1 + \frac{k}{r\Omega}\right)b = \left(\nu - \frac{\mu k}{r}\right)\phi\,, \\
\frac{1}{\sigma\Omega}b - \frac{1}{a^{\mathrm{Le}z_1}}F - \frac{1}{a^{\mathrm{Le}z_2}}G + \frac{1}{a^{\mathrm{Le}z_2}}H - s = \frac{1}{\sigma\Omega}\phi\,, \\
\frac{1}{\sigma\Omega}\frac{1-\mathrm{Le}}{\mathrm{Le}}b + \frac{z_3}{a^{z_3}}C + \frac{z_4}{a^{z_4}}D + \frac{z_1}{a^{\mathrm{Le}z_1}}F + \frac{z_2}{a^{\mathrm{Le}z_2}}G - \frac{z_2}{a^{\mathrm{Le}z_2}}H - \frac{1-\mathrm{Le}}{\mathrm{Le}}s = \left(\frac{1}{\sigma\Omega}\frac{1-\mathrm{Le}}{\mathrm{Le}} - \Gamma\right)\phi\,, \\
\frac{1}{\sigma\Omega\,\mathrm{Le}}b + \frac{z_3}{a^{z_3}}C + \frac{z_4}{a^{z_4}}D - \frac{1}{a^{\mathrm{Le}(z_2+1)}}\frac{k}{\delta}H - \frac{1}{\mathrm{Le}}s = \left(\frac{1}{\sigma\Omega\,\mathrm{Le}} - \frac{k\Gamma\theta_b^0}{\Delta} + 1 - \nu\right)\phi\,, \\
\frac{1}{\sigma\Omega\,\mathrm{Le}}b + \frac{1}{a^{z_3}}C + \frac{1}{a^{z_4}}D - \frac{1}{\mathrm{Le}}s = \frac{1}{\sigma\Omega\,\mathrm{Le}}\phi\,,
\end{gathered} \tag{8}$$

where the notation $\Delta/(\Delta + l) = \delta$ is introduced.

In the case of investigating the combustion stability in a rocket-propellant engine, it is necessary, within the framework of the approximation used here, to supplement this system with an equation which can be derived from the condition of balance of the gas mass in the combustion chamber [10]. Here we will assume that the pressure deviation ϕ from the stationary value is assigned [5–8].

3. Determination of the Region of Stable Combustion of a Propellant. The system of equations (8) can be used to find the condition of stable combustion of a propellant that is in open space (then $\eta = 1$). The possible instability of combustion is caused here only by the nature of the propellant itself. And if in Eq. (8) we set $\phi = 0$, then a homogeneous system will be obtained. The equality to zero of its determinant is the condition of solvability of this system. The behavior of disturbances, whether they will increase infinitely, decrease, or remain limited in absolute value depending on time, is determined by the form of the increment Ω that is generally a complex quantity. If the real part of Ω is equal to zero, then the solutions of (5) of the form (6) will be periodic with limited amplitudes. When these solutions are realized, the values of the physical parameters determine the boundary between the stable and unstable regimes of combustion.

The condition of equality to zero for the determinant of the homogeneous side of Eqs. (8) leads to the equation

$$W'\Phi + V\Psi + \frac{\delta a^{\mathrm{Le}}}{k} H\,(\sigma\Omega W' + z_1 V) + P\left(1 - \frac{\delta a^{\mathrm{Le}}}{k} z_1\right) = 0\,, \tag{9}$$

$$W' = \delta\,(z-1)\left(\frac{1}{\Omega} + \frac{r}{k}\right) + 1 - \delta = \delta\,(z-1)\,\frac{r}{k} + W'\,,\quad W' = \delta\,\frac{z-1}{\Omega} + 1 - \delta\,,\quad V = 1 + \sigma\Omega\delta\,\frac{r}{k}\,,$$

$$P = z_\Delta\,(z_3 - z_4)\,,\quad H = z_\Delta a^{z_2\mathrm{Le}}\,(a^{-z_4} - a^{-z_3})\,,\quad z_\Delta = z_1 - z_2\,,$$

$$\Phi = (z_4 a^{-z_4} - z_3 a^{-z_3})\,(z_1 a^{z_1\mathrm{Le}} - z_2 a^{z_2\mathrm{Le}}) + \sigma\Omega\,(a^{-z_4} - a^{-z_3})\,(a^{z_1\mathrm{Le}} - a^{z_2\mathrm{Le}})\,,$$

$$\Psi = (z_4 a^{-z_4} - z_3 a^{-z_3})\,(a^{z_2\mathrm{Le}} - a^{z_1\mathrm{Le}}) + (a^{-z_4} - a^{-z_3})\,(z_2 a^{z_1\mathrm{Le}} - z_1 a^{z_2\mathrm{Le}})\,.$$

When Le = 1, expression (9) is simplified:

$$\frac{k}{a} z_\Delta\left[1 - (z-1)\left(\frac{r}{k} + \frac{1}{\Omega}\right)\right] - z_\Delta z_1 - (a^{-z_\Delta} - 1)\,\left[(z - z_2)\left(\frac{z}{k}\,\sigma\Omega\delta + 1\right) + (z-1)\,(\sigma\delta - 1) + \sigma\Omega\,(1-\delta)\right] = 0\,.$$

This equation was previously obtained in [6]; its properties are also given partially in this work.

At the stability boundary, $\Omega = i\omega$ is a pure imaginary quantity, where ω has the meaning of the oscillation frequency of the diffusion-thermal and hydrodynamic parameters. With such a form of the increment Ω, Eq. (9) contains real and imaginary parts, each of which separately must be equal to zero. From the two equations obtained in this way, we can determine the boundary of the region of stable combustion in the form $k = k(r, \delta, \sigma, \mathrm{Le}, a)$, eliminating the frequency ω from the resultant dependences $k = k(\omega, \delta, \sigma, \mathrm{Le}, a)$ and $r = r(\omega, \delta, \sigma, \mathrm{Le}, a)$; these dependences parametrically define k as a function of r and some other physical constants entering into Eq. (9).

Omitting the details of the calculation, we note that the evaluations are substantially simplified owing to the visual and compact representation if we introduce operators for work with cumbersome complex expressions. Suppose that $\alpha = \alpha_1 + i\alpha_2$ and $\beta = \beta_1 + i\beta_2$ are arbitrary complex numbers. Then

$$\alpha\cdot\beta = \begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}_- + i\begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}_+\,,$$

where the operators $[\ ]_-$ and $[\ ]_+$ mean

$$\begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}_- = \alpha_1\beta_1 - \alpha_2\beta_2\,,\quad \begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}_+ = \alpha_1\beta_2 + \alpha_2\beta_1\,.$$

Here and below subscripts 1 and 2 are assigned to the real and imaginary parts of the quantities W', V, P, and others. We also introduce the notation

$$N = \mathrm{Re}\, z_1 - \sigma\omega W_2' + i\,(\mathrm{Im}\, z_1 + \sigma\omega W_1')\,,\quad K = \mathrm{Im}\,(z + z_1) + i\,[1 - \mathrm{Re}\,(z + z_1)]\,.$$

In the end, the boundary of the stable-combustion region is determined in the following manner:

(1) we solve a quadratic equation relative to the unknown quantity r/k:

$$A_3\left(\frac{r}{k}\right)^2 + B_3\left(\frac{r}{k}\right) + C_3 = 0\,, \tag{10}$$

$$\frac{A_3}{\sigma\omega\delta^2}=\begin{bmatrix}H_1 & H_2\\ K_1 & K_2\end{bmatrix}_-\left(\begin{bmatrix}\operatorname{Re}z & \operatorname{Im}z\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +\sigma\omega\Psi_1\right)-\begin{bmatrix}H_1 & H_2\\ K_1 & K_2\end{bmatrix}_+\left(\begin{bmatrix}\operatorname{Re}z & \operatorname{Im}z\\ \Phi_1 & \Phi_2\end{bmatrix}_- -\sigma\omega\Psi_2\right),$$

$$B_3=\delta\left(\begin{bmatrix}\operatorname{Re}z & \operatorname{Im}z\\ \Phi_1 & \Phi_2\end{bmatrix}_- -\sigma\omega\Psi_2\right)\left(\begin{bmatrix}H_1 & H_2\\ N_1 & N_2\end{bmatrix}_+ -\begin{bmatrix}\operatorname{Re}z_1 & \operatorname{Im}z_1\\ P_1 & P_2\end{bmatrix}_+\right)-$$

$$-\sigma\omega\delta\begin{bmatrix}H_1 & H_2\\ K_1 & K_2\end{bmatrix}_+\left(\begin{bmatrix}W_1' & W_2'\\ \Phi_1 & \Phi_2\end{bmatrix}_- +P_1+\Psi_1\right)-$$

$$-\delta\left(\begin{bmatrix}\operatorname{Re}z & \operatorname{Im}z\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +\sigma\omega\Psi_1\right)\left(\begin{bmatrix}H_1 & H_2\\ N_1 & N_2\end{bmatrix}_- -\begin{bmatrix}\operatorname{Re}z_1 & \operatorname{Im}z_1\\ P_1 & P_2\end{bmatrix}_-\right)+$$

$$+\sigma\omega\delta\begin{bmatrix}H_1 & H_2\\ K_1 & K_2\end{bmatrix}_-\left(\begin{bmatrix}W_1' & W_2'\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +P_2+\Psi_2\right),$$

$$C_3=\left(\begin{bmatrix}W_1' & W_2'\\ \Phi_1 & \Phi_2\end{bmatrix}_- +P_1+\Psi_1\right)\left(\begin{bmatrix}H_1 & H_2\\ N_1 & N_2\end{bmatrix}_+ -\begin{bmatrix}\operatorname{Re}z_1 & \operatorname{Im}z_1\\ P_1 & P_2\end{bmatrix}_+\right)-$$

$$-\left(\begin{bmatrix}W_1' & W_2'\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +P_2+\Psi_2\right)\left(\begin{bmatrix}H_1 & H_2\\ N_1 & N_2\end{bmatrix}_- -\begin{bmatrix}\operatorname{Re}z_1 & \operatorname{Im}z_1\\ P_1 & P_2\end{bmatrix}_-\right);$$

(2) from this quadratic equation we establish the dependence $r/k = r/k(\omega)$ as a function of the frequency ω and then, considering this function simultaneously with the formula given below,

$$\frac{k}{\delta a^{\mathrm{Le}}}=\frac{\begin{bmatrix}\operatorname{Re}z_1 & \operatorname{Im}z_1\\ P_1 & P_2\end{bmatrix}_+ +\sigma\omega\delta\frac{r}{k}\begin{bmatrix}H_1 & H_2\\ K_1 & K_2\end{bmatrix}_+ -\begin{bmatrix}H_1 & H_2\\ N_1 & N_2\end{bmatrix}_+}{\delta\frac{r}{k}\begin{bmatrix}\operatorname{Re}z-1 & \operatorname{Im}z\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +\begin{bmatrix}W_1' & W_2'\\ \Phi_1 & \Phi_2\end{bmatrix}_+ +\begin{bmatrix}1 & \sigma\omega\delta\frac{r}{k}\\ \Psi_1 & \Psi_2\end{bmatrix}_+ +P_2},$$

we find the sought boundary of the stability region.

Both roots of Eq. (10) are real. They are numbered by subscripts 1 and 2 depending on the chosen sign + or sign – before the square root of the discriminant (10): r_1/k_1 and r_2/k_2. Since these roots lead to

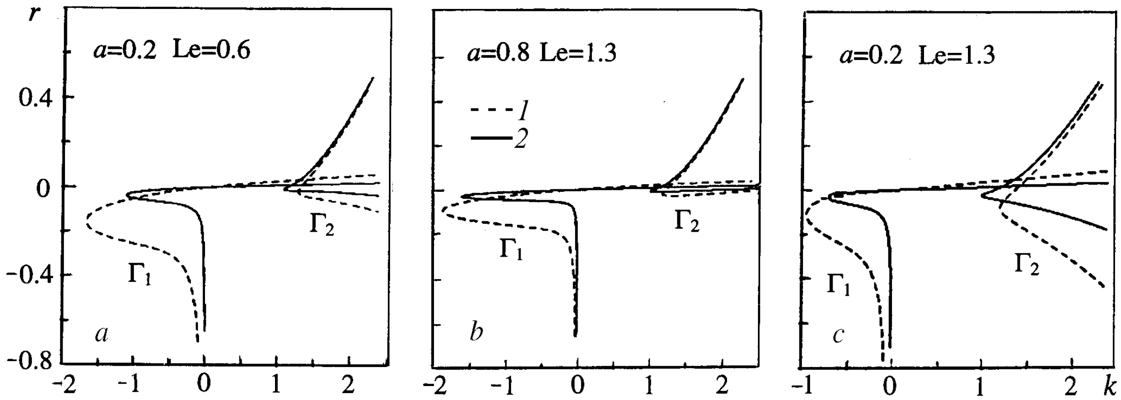

Fig. 2. Change in curves Γ_1 and Γ_2 that determine the boundary of the regions of stable and unstable combustion as a function of the parameters a, Le, and σ: 1) σ = 0.01 and 2) 0.001. The qualitative behavior of these regions as a function of the parameter a can be tracked in Fig. 2b and c, as a function of the Le number in Fig. 2a and c, and as a function of σ in Fig. 2a, b, and c.

two regions of stable combustion, the intersection of these regions corresponds to the solution of the problem formulated here.

Some results of the calculation according to the above algorithm are presented in Fig. 2. The numbering of curves Γ_1 and Γ_2 corresponds to that of the roots in Eq. (10). As the base values, we take the numerical values of the physical parameters given in [10] for powder H (powder H is a ballistic fuel which contains 56.44% colloxylin, 27.72% nitroglycerin, 10.89% dinitrotoluene, 2.97% dimethyl-diphenylurea, 0.99% vaseline, and 0.99% water by weight) at a pressure of p_0 = 50 atm and an initial temperature of T_0 = 293 K. Further parametric analysis was made in the neighborhood of these data of the form: Q = 3347 kJ/kg, c_c = 3.2 kJ/(kg·K), $\rho_c = 1.6 \cdot 10^3$ kg/m^3, λ_c = 0.383 W/(m·K), L = –494 kJ/kg, T_s^0 = 673 K, and $u^0 = 6.7 \cdot 10^{-3}$ m/sec. In addition to these data we also take $D \sim 2.3 \cdot 10^{-5}$ m^2/sec, γ = 1.4, $\rho^0 \sim 0.8$ kg/m^3, and $\lambda^0 \sim 0.06$ W/(m·K).

The stability region denoted by curves Γ_2 is located on the left of them when the r axis moves in the positive direction [6, 10, 11]. The stability region can quite easily be determined from curves Γ_1 if in Eq. (9) we assume that Le = 1 and expand it in the parameter $\sigma\Omega << 1$ up to the terms linear in $\sigma\Omega$:

$$1 - k + (z - 1)\left(r + \frac{k}{\Omega}\right) + \sigma\Omega\left[2\ln\frac{1}{a} + 3a + (1 - a)\left(W' + 1 + \delta\frac{r}{k}\right) + \frac{2k}{\delta}(W' - 1)\right] = 0\,.$$

Now we let $r \to \infty$ considering k to be limited. This will correspond to small Ω if the values of k lie near the boundary of stable combustion. Discarding the small terms of the form $O(1/r)$ and $O(\Omega^2)$, after simple calculation we find

$$\Omega = -\frac{k + \sigma\delta(1 - a)}{4\sigma k}\,.$$

Whence it is evident that the combustion is stable, provided that $k > -\sigma\delta(1 - a)$. Thus, when the r axis moves along the boundary Γ_1 in the positive direction, the region of stable combustion will be located on the right. (We can confirm this conclusion by a numerical analysis of (9). For this, the equality of Eq. (9) to zero at an arbitrary point of the plane k, r must be taken with the nonzero real part Ω. This analysis can most easily be made near the stability boundary.)

In practice, the parameters k and r are positive. In the case of their regularity, we come to the following conclusions: if the pyrolysis reaction on the propellant surface is exothermal, then the combustion is stable within very narrow limits (see Fig. 2) of change of k and r (this is not the case if the thermal effect is rather small; for this see below); moreover, it is required that $r << 1$ and $k \approx 1$. On the contrary, propellants with an endothermal reaction of pyrolysis have a wider region of stable combustion.

We can generalize the results of the analysis of Eq. (9) by drawing the conclusion that account for the dimensionless time σ of relaxation of the gas phase leads to results which differ from the theory with $\sigma = 0$ not only quantitatively but also qualitatively. This feature is retained for an arbitrarily small σ. For example, in the exothermal reaction of pyrolysis of the propellant and in the limit $\sigma \to 0$ the combustion with positive values of k and r is unstable (practically already for $\sigma < 0.05$), but in the endothermal reaction of pyrolysis the combustion is stable. At the same time, the theory with $\sigma = 0$ formulates the stability condition for combustion by the inequality [10, 14]

$$r > \frac{(k-1)^2}{k+1}$$

or

$$\frac{2 + r - \sqrt{r\,(8+r)}}{2} < k < \frac{2 + r + \sqrt{r\,(8+r)}}{2}\,,$$

whence, however, it is evident that the stable combustion of propellants, for which $r = 0$, is impossible, whereas in the theory with a constant temperature of the decomposition surface of a propellant ($r = 0$) the region of stable combustion is given by the condition $k < 1$ [12].

The above contradictions are evidence in favor of the fact that in the theory of nonstationary combustion of powders and propellants it is necessary initially to consider the most rapid processes. Slow processes (for example, heat transfer in the solid phase) must be taken into account as the secondary factors that weakly disturb the nonstationary regime of combustion determined mainly by the gas phase and the chemical reaction. The established situation can easily be understood from the representation of the unstable combustion of propellants as a dynamic system [10, 13]. Within the framework of B. V. Novozhilov's theory [14] ($\sigma = 0$) and in the linear approximation, the nonstationary portion of the surface temperature of decomposition of the propellant satisfies the ordinary differential equation of second order with an oscillation decrement Λ and a natural frequency ω_0 [13]

$$\Lambda = \frac{r\,(k+1) - (k-1)^2}{r^2}\,, \quad \omega_0 = \frac{\sqrt{k}}{r}\,.$$

For $r \to 0$ we have the equation with a vanishingly small parameter at a higher derivative [13] which cannot be ignored for the reasons of smallness of this parameter. Therefore, the results of Ya. B. Zel'dovich's theory [12] do not follow as a particular case from B. V. Novozhilov's theory [14].

Based on the investigation carried out here, it can also be assumed that on the surface of decomposition of homogeneous solid propellants, usually we have either the endothermal $L > 0$ or the weakly exothermal $|L| << 1$ (more exactly, for $\delta < 1.2$–1.3) reaction of pyrolysis as the most stable process with respect to high-frequency oscillations. It is precisely for these cases that curve Γ_1 is similar to a hyperbola and remains in the third quadrant of the plane k, r or makes a small loop into the fourth quadrant without intersecting curve Γ_2 and returning again to the third quadrant.

On the decomposition surface of the solid phase of the powder H, the reaction is exothermal. The point of intersection of Γ_1 and Γ_2 corresponds here to frequencies of $\omega \sim 1/\sigma$ or higher. For this powder we have: θ_0 = 0.435, l = –0.228, and δ = 1.68. Therefore, powder H, for which $r \ll 1$ and $k \approx 1$, must possess weak stability with respect to high-frequency oscillations.

Account for the inertia of the chemical-reaction zone, primarily in the solid phase, needed at high frequencies ω can change this situation toward the extension of the region of stable combustion for exothermal reactions of pyrolysis. But, as is seen, for this mechanism of decomposition of the propellant account for increasingly more rapid processes can lead to unexpected conclusions. This uncertainty is not observed when the powder is decomposed by the mechanism of evaporation [4].

The increase in the Lewis number results in a decrease in the region of stable combustion. The influence of Le is here manifested only very slightly for $|r| > 1$, i.e., at low natural frequencies ω. The small deviation of the value of Le from unity affects strongly the size of the stability region at high frequencies: for the boundaries Γ_1 — with $|r| \ll 1$ and $k > 0.5$, while for the boundaries Γ_2 — with $r \ll 1$ and $k > 3$. Moreover, the parameter a varies within the limits [0.2, 0.8], whereas the parameter δ varies within [0.25, 2.5].

When $\sigma\Omega \ll 1$, which is fulfilled at relatively low natural frequencies of the combined system solid propellant–gas phase, we have expansions with an accuracy to $O(\sigma^2\Omega^2)$:

$$(z_4 a^{-z_4} - z_3 a^{-z_3})\,(z_1 a^{z_1 \mathrm{Le}} - z_2 a^{z_2 \mathrm{Le}}) \approx -a^{\mathrm{Le}-1} - \sigma\Omega\left[\frac{1}{a} + a^{\mathrm{Le}-1} + a^{\mathrm{Le}}\,\mathrm{Le}\left(1 + \frac{1}{a}\right)\right],$$

$$(a^{-z_4} - a^{-z_3})\,(a^{z_1 \mathrm{Le}} - a^{z_2 \mathrm{Le}}) \approx \left(1 - \frac{1}{a}\right)(a^{\mathrm{Le}} - 1) - 2\,\mathrm{Le}\,\sigma\Omega\left(a^{\mathrm{Le}} - \frac{1}{a}\right)\ln\frac{1}{a},$$

$$(z_4 a^{-z_4} - z_3 a^{-z_3})\,(a^{z_2 \mathrm{Le}} - a^{z_1 \mathrm{Le}}) \approx \frac{a^{\mathrm{Le}} - 1}{a} + \sigma\Omega\,\mathrm{Le}\left[(a^{\mathrm{Le}} - 1)\left(1 + \frac{1}{a}\right) - \frac{2}{a}\ln\frac{1}{a}\right],$$

$$(a^{-z_4} - a^{-z_3})\,(z_2 a^{z_1 \mathrm{Le}} - z_1 a^{z_2 \mathrm{Le}}) \approx \frac{1}{a} - 1 - \sigma\Omega\left[(a^{\mathrm{Le}} + 1)\left(1 - \frac{1}{a}\right) - \frac{2}{a}\,\mathrm{Le}\ln\frac{1}{a}\right],$$

$$z_\Delta a^{z_2 \mathrm{Le}}\,(a^{-z_4} - a^{-z_3}) \approx 1 - \frac{1}{a} + 2\sigma\Omega\left(1 - \frac{1}{a} - \frac{\mathrm{Le}}{a}\ln\frac{1}{a}\right),$$

$$z_\Delta\,(z_3 - z_4)\left(\frac{k}{\delta a^{\mathrm{Le}}} - z_1\right) \approx \frac{k}{\delta a^{\mathrm{Le}}} - 1 + \sigma\Omega\left[2\,(\mathrm{Le} + 1)\left(\frac{k}{\delta a^{\mathrm{Le}}} - 1\right) - 1\right].$$

Using the above formulas and performing simple but unwieldy calculations, we can obtain the equation for the boundary Γ_2 in the form

$$r = \frac{(k-1)^2}{k+1}\left[1 + \sigma\,\frac{k}{k+1}\,(F_1 - F_2) + O\,(\sigma^2\omega^2)\right],$$

$$F_1 = \frac{k+1}{k-1}\,a^{1-\mathrm{Le}}\left(\frac{1}{k-1}\,f_1 + 1 - a^{\mathrm{Le}-1}\right) - g\,\frac{k-3}{(k-1)^2},$$

$$F_2 = \frac{2k}{k-1}\,a^{1-\mathrm{Le}}\left(1 - a^{\mathrm{Le}-1} + \frac{1}{2\delta}\,\frac{k+1}{k-1}\,f_2\right),$$

$$g = \delta\,(1-a)\,, \quad f_1 = 1 + a^{\mathrm{Le}}\left[\mathrm{Le}\left(\frac{1}{a} - 1\right) - \frac{2}{a}\left(1 + \mathrm{Le}\ln\frac{1}{a}\right)\right], \quad f_2 = \frac{\mathrm{Le} - 1}{a} + a^{\mathrm{Le}-1} - \mathrm{Le}\,.$$

Whence at Le = 1 we obtain the formula from [6]

$$r = \frac{(k-1)^2}{k+1}\left[1 - \sigma \frac{k}{(k-1)^2}\left(a - 2\ln a + g\frac{k-3}{k+1}\right) + O(\sigma^2\omega^2)\right].$$

For the boundary Γ_1 a similar formula looks much simpler:

$$k = -\sigma\delta(1-a)\frac{(r-1)(2r-1)}{2r^2} + O(\sigma^2\omega^2), \quad r < 0.$$

For r which are small in absolute value, the above approximate formulas become unacceptable for determination of the boundary of combustion stability. Some interest in the negative values of r has appeared in connection with [15], in which it is suggested that $r < 0$ is possible for propellants decomposed by the evaporation mechanism. As far as $k < 0$ is concerned, daily experience suggests that such values of this parameter r are impossible. But, on the other hand, the question arises "why, stable combustion is possible, isn't it?" There can be two answers: either our practice is still limited for observing the processes of combustion with $k < 0$ or the combustion model considered in this work can give nonphysical regions of stable combustion.

4. Nonstationary Rate of Combustion at Variable Pressure. Acoustic Admittance. The characteristic of unstable combustion of primary practical importance is the decomposition rate b of the propellant. All the remaining quantities, i.e., the temperature, the concentration of the reagent, the acoustic admittance, etc. can be expressed in terms of b. Amplification of acoustic oscillations in the combustion chambers of propellant rocket engines leads to an undesirable effect, i.e., acoustic instability. The modulus of the ratio between the amplitudes of the sound wave reflected from the combustion surface of the propellant to the sound wave incident on it is determined by the real part of the acoustic admittance ζ. If Re $\zeta < 0$, then the sound waves in the absence of mechanical dissipative processes will be amplified [10].

To simplify the calculation of the acoustic admittance, from the fifth, sixth, and seventh equations of system (8) we find H/ϕ:

$$\frac{Ha^{-z_2\mathrm{Le}}}{\phi} = \frac{\delta a^{\mathrm{Le}}}{k - z_1\delta a^{\mathrm{Le}}}\left(\Gamma + \frac{k\Gamma\theta_\mathrm{b}^0}{\Delta} + 1 - \nu - \frac{Fz_2 a^{-z_1\mathrm{Le}}}{\phi} - \frac{Gz_1 a^{-z_2\mathrm{Le}}}{\phi}\right). \tag{11}$$

Let us now consider the hydrodynamic part of the problem. The equations of continuity (the second and fifth ones in Eq. (1)) with the use of the equation of state of the gas after the transition to the Lagrangian coordinate and dimensionless quantities can be written in the form

$$\sigma\frac{\partial}{\partial\tau}\frac{\theta}{\eta} + B\frac{\partial}{\partial\xi}\frac{\theta}{\eta} = \theta_\mathrm{b}^0\frac{\partial}{\partial\xi}W_\mathrm{g}, \quad \sigma\frac{\partial}{\partial\tau}\frac{\theta_\mathrm{b}}{\eta} + B\frac{\partial}{\partial\xi}\frac{\theta_\mathrm{b}}{\eta} = \theta_\mathrm{b}^0\frac{\partial}{\partial\xi}W_\mathrm{g.b}, \tag{12}$$

where

$$W_\mathrm{g} = \frac{v+u}{v_\mathrm{b}^0 + u^0}, \quad W_\mathrm{g.b} = \frac{v_\mathrm{b}+u}{v_\mathrm{b}^0 + u^0}.$$

The gas velocities W_g and $W_\mathrm{g.b}$ must satisfy the boundary conditions

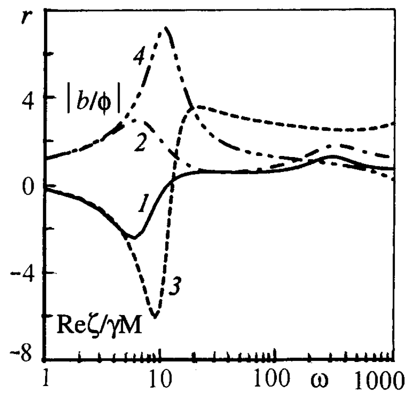

Fig. 3. Dependences of the modulus of the relative combustion rate b/ϕ (curves 2, 4) and the real part of the acoustic admittance ζ (curves 1, 3) in units of γ M as functions of the oscillation frequency ω of the pressure.

$$\xi = 0\,, \quad W_{\mathrm{g}} = \frac{\theta}{\theta_{\mathrm{b}}^0}\frac{B}{\eta}\,; \quad \xi = \xi_{\mathrm{f}}^0\,, \quad W_{\mathrm{g}} = W_{\mathrm{g.b}}\,.$$

In the stable regime of combustion we have

$$W_{\mathrm{g}}^0 = \frac{\theta^0}{\theta_{\mathrm{b}}^0}\,, \quad W_{\mathrm{g.b}}^0 = 1\,.$$

Solving Eqs. (12) simultaneously with the above-given boundary conditions in the linear approximation with the representations of the velocities W_{g} and $W_{\mathrm{g.b}}$ of the gas given below

$$W_{\mathrm{g}} = W_{\mathrm{g}}^0 + w_{\mathrm{g}}\phi\,\exp\,(\Omega\tau)\,, \quad W_{\mathrm{g.b}} = W_{\mathrm{g.b}}^0 + w_{\mathrm{g.b}}\phi\,\exp\,(\Omega\tau)\,,$$

we find the response $w_{\mathrm{g.b}}$ of the gas velocity in the flame front:

$$w_{\mathrm{g.b}} = w_{\mathrm{g}}\,(\xi = \xi_{\mathrm{f}}^0) = \frac{\sigma\Omega}{\theta_{\mathrm{b}}^0}\,I_1 + \frac{b}{\phi} + \frac{1}{\theta_{\mathrm{b}}^0\phi}\,\vartheta_{\mathrm{b}}\,(\xi = \xi_{\mathrm{f}}^0) - 1\,, \quad I_1 = \int\limits_0^{\xi_{\mathrm{f}}^0}\left(\frac{\vartheta}{\phi} - \theta^0\right)d\xi\,.$$

The basic formula for the acoustic admittance ζ is given in [8]. In the notation taken here it is as follows:

$$\frac{\zeta}{\gamma \mathrm{M}} = -\,w_{\mathrm{g.b}} + z_2\left[\frac{1}{\theta_{\mathrm{b}}^0\phi}\,\vartheta_{\mathrm{b}}\,(\xi = \xi_{\mathrm{f}}^0) - \Gamma\right], \quad \mathrm{M} = (v_{\mathrm{b}}^0 + u^0)/c_0\,. \tag{13}$$

In this form the formula is independent of the mechanism of decomposition of the propellant and of the magnitude of the Le number. The distinction will manifest itself in using a particular form of the quantities $w_{\mathrm{g.b}}$ and $\vartheta_{\mathrm{b}}(\xi_{\mathrm{f}}^0)$.

Having evaluated the integral I_1 and substituted ϑ_{b} (from Sec. 2) and $w_{\mathrm{g.b}}$ into Eq. (13), we can give the following form to the expression for ζ:

$$\frac{\zeta}{\gamma \mathrm{M}} = 1 - \frac{b}{\phi} - \frac{q z_1}{\theta_b^0} \frac{H a^{-z_2 \mathrm{Le}}}{\phi} - \frac{\sigma\Omega}{\theta_b^0} I_1 - \Gamma \,,$$

$$I_1 = (a^{-z_1 \mathrm{Le}} - 1) \frac{q}{z_1} \frac{F}{\phi} + (a^{-z_2 \mathrm{Le}} - 1) \frac{q}{z_2} \frac{G}{\phi} + \frac{I_2}{\sigma\Omega} + I_3 \,,$$

$$I_2 = (\Delta + l)\,(a^{-\mathrm{Le}} - 1) \left(1 + \sigma\Omega\Gamma - \frac{b}{\phi}\right), \quad I_3 = (\Gamma + 1)\,(\theta_0 - l)\ \mathrm{Le} \ln \frac{1}{a} - (\Delta + l)\,(a^{-\mathrm{Le}} - 1) \,.$$

Thus, knowing b/ϕ, we can also find the acoustic admittance. The parameters F/ϕ and G/ϕ are explicitly expressed in terms of b/ϕ from the first three equations of system (8). The response modulus b/ϕ of the decomposition rate of the propellant can be calculated analytically or numerically by solution of Eqs. (8). The calculation results are given in Fig. 3. The dimensionless numerical parameters are equal to $\sigma = 10^{-2}$, $l = -0.1$, $r = 0.1$, $\nu = 0.8$, $\mu = 0.2$, and $\vartheta_0 = 0.4$. Curves 1 and 2 are constructed for $k = 1.4$, while curves 3 and 4 are constructed for $k = 1.3$. The Lewis number for all the curves is equal to 1.5.

The amplification of acoustic oscillations occurs when the real part ζ is negative [2, 10, 11]. The analysis of the acoustic admittance performed in the present work has shown that Re $\zeta < 0$ if the combustion process occurs near the stability boundary (in Fig. 3, the coefficients k and r are taken near this boundary) and if the oscillation frequency of the pressure is close to the natural frequency of the system solid propellant–gas phase. The closer the oscillation frequency to the boundary, the larger Re ζ in absolute value. This conclusion, drawn from the previous works on nonstationary combustion in which the relaxation time of the gas phase was disregarded, also remains valid for the more extended model considered in the present work. Therefore, the above-stated results of the influence of physical parameters on the size of the region of stable combustion can readily be extended to the problem of amplification of acoustic waves by a burning propellant.

The influence of the Lewis number on both the nonstationary combustion rate and the acoustic admittance is pronounced only at frequencies of the order of the reciprocal of the relaxation time in the gas phase or higher.

The numerical solution of system (8) has been carried out using the IMSL Fortran Powerstation application package.

NOTATION

κ_c, thermal diffusivity of the propellant; B, dimensionless linear combustion rate of the propellant; a, burning out; $\vartheta_c(\xi)$, $\vartheta(\xi)$, and $\vartheta_b(\xi)$, spatial components of the disturbance of the dimensionless temperature in the propellant, in the heating zone, and the flame zone, respectively; $y(\xi)$, spatial component of the disturbance of the concentration Y; s and b, disturbance amplitudes of the position of the flame front and the combustion rate, respectively; ρ_c and ρ, densities of the propellant and the gas; λ_c and λ, coefficients of thermal conductivity of the propellant and the gas, respectively; c_c, heat capacity of the propellant; c_p, heat capacity of the gas at constant pressure; Y, mass concentration of the combustible component; v and v_b, velocities of the gas; γ, adiabatic exponent; m_1 and m_2, mass rates of combustion in the solid and gas phases; M, Mach number; c_0, velocity of sound. Subscripts: g, gas; c, condensed; s, surface; f, flame; p, pressure; b, burning.

REFERENCES

1. B. V. Orlov and G. Yu. Mazing, *Thermodynamic and Ballistic Principles of Designing of Solid-Propellant Rocket Engines* [in Russian], Moscow (1979).

2. F. A. Williams, *Combustion Theory* [Russian translation], Moscow (1971).
3. K. K. Kuo, *Principles of Combustion*, Wiley, New York (1986).
4. B. V. Novozhilov, *Khim. Fiz.*, **7**, No. 3, 388–396 (1988).
5. B. V. Novozhilov, *Khim. Fiz.*, **8**, No. 1, 102–111 (1989).
6. V. N. Vilyunov and K. O. Sabdenov, *Khim. Fiz.*, **11**, No. 3, 415–423 (1992).
7. Z. I. Kaganova, *Khim. Fiz.*, **12**, No. 6, 866–873 (1993).
8. B. V. Novozhilov, *Khim. Fiz.*, **10**, No. 11, 1518–1523 (1991).
9. T. K. Sherwood, R. L. Pigford, and C. R. Wilke, *Mass Transfer* [Russian translation], Moscow (1982).
10. B. V. Novozhilov, *Nonstationary Combustion of Solid Propellants* [in Russian], Moscow (1973).
11. Ya. B. Zel'dovich, O. I. Leipunskii, and V. B. Librovich, *Theory of Nonstationary Combustion of Powder* [in Russian], Moscow (1975).
12. Ya. B. Zel'dovich, *Zh. Éksp. Teor. Fiz.*, **12**, 498–527 (1942).
13. K. O. Sabdenov, *Khim. Fiz.*, **12**, No. 3, 419–424 (1993).
14. B. V. Novozhilov, *Prikl. Mekh. Tekh. Fiz.*, No. 4, 157–160 (1955).
15. L. K. Gusachenko and V. E. Zarko, in: *Proc. All-Russian Sci. Conf. "Fundamental and Applied Problems of Modern Mechanics"* [in Russian], Tomsk (1998), pp. 53–54.